

# Phase coherence in one dimensional superconductivity by power-law hopping

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A central result in one-dimensional (1D) superconductivity is that even at zero temperature quantum fluctuations destroy phase coherence. Here we put forward a mechanism which can restore phase coherence: power-law hopping. We study a 1D attractive- $U$  Hubbard model with power-law hopping by Abelian bosonization and density-matrix renormalization group (DMRG) techniques. The parameter that controls the hopping decay acts as the effective, non-integer spatial dimensionality  $d_{\text{eff}}$ . We show analytically that for any  $d_{\text{eff}} > 1$  at zero temperature, power-law hopping suppresses fluctuations and induces phase coherence, namely, long-range superconducting order. A detailed DMRG analysis fully supports these findings. These results are also of direct relevance to quantum magnetism as our model can be mapped onto a spin-chain with power-law decaying couplings, which can be studied experimentally by cold ion-trap techniques.

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According to the Mermin-Wagner-Hohenberg theorem [1, 2] quantum and thermal fluctuations in low dimensions prevent the spontaneous breaking of a continuous symmetry. A paradigmatic example is a one-dimensional (1D) superconductor (SC), where fluctuations of the SC order parameter result in quasi long-range order at zero temperature, i.e., the algebraic decay of the order parameter correlation function [3]. By contrast superconducting long-range order (LRO), equivalent to phase coherence in this context, occurs if the correlation function does not decay even for arbitrarily large distances.

Quantum phase-slip (QPS) fluctuations (i.e., processes where the amplitude of the order parameter temporarily vanish in some region along the wire, allowing its phase to slip by  $2\pi$  [4–6]) play an especially important role, as they are believed to destroy quasi-LRO and induce a finite resistivity at low temperatures observed in SC wires whose diameter is much smaller than the SC coherence length [cf. Refs. [7, 8] and references therein]. Interestingly, recent theoretical works have shown the possibility to stabilize a 1D SC through the suppression of QPS fluctuations. This is achieved by a weak coupling of the wire to a dissipative environment (such as a weakly disordered metallic thin film [9, 10], a metallic dirty lead [11], a graphene sheet [12], or two dissipative electrodes attached to the ends of the SC wire [13, 14]) that suppresses fluctuations, including QPS, and induce LRO [15]. Experimentally, restoration of phase coherence has been recently observed in thin Zn [16, 17] and Al [18] nanowires by increasing the coupling of the wire to dissipative electrodes. Therefore one of the main theoretical challenges in the field is to identify mechanisms that are capable to inhibit the proliferation of QPS and restore phase coherence in 1D. The increase of the effective spa-

tial dimensionality is an appealing choice. Interestingly, in the context of non-interacting 1D weakly disordered systems [19, 20], it is well known that power-law hopping  $\propto 1/|i-j|^\alpha$  (with  $\alpha > 1/2$ ) effectively mimics the properties of a system in  $d_{\text{eff}} = 2/(2\alpha - 1)$  spatial dimensions with short-range hopping. This effect seems to be robust to the presence of interactions [21].

The achievement of phase coherence in low dimensional SC is also relevant for applications: from the miniaturization of the SC circuits to the enhancement of the critical temperature in SC nanostructures and thin films [22–25].

In this Letter we study the stabilization of a 1D SC by power-law single-particle hopping. We focus our study on the 1D attractive- $U$  Hubbard model with algebraically decaying hopping  $t_{lm} \propto t/|l-m|^\alpha$ , where  $\alpha$  is the parameter controlling the decay. We study the quantum phases of the system at zero temperature by analytical (Abelian bosonization and a variational approach) and numerical density-matrix renormalization group (DMRG) techniques. Our main result is the restoration of LRO at zero temperature for  $\alpha < 3/2$ , corresponding to  $d_{\text{eff}} > 1$ . Algebraic coupling occurs in a variety of physical systems, such as Josephson junction arrays [26], materials with strong dipolar interactions [27], and atoms in cavities realizing effectively quantum spin chains with long-range exchange interactions [28, 29]. In the latter, a spin-dependent optical dipole force applied to a cold atom gas makes possible to engineer power-law anti-ferromagnetic interactions with  $0 \leq \alpha \leq 3$  [28, 29]. As we show below, spin chains with power-law exchange can be mapped onto a 1D SC with power-law hopping, so the ideas and techniques we introduce here are of direct relevance for these problems as well.

*Model.-* We study the  $L$ -site spin-1/2 1D Hubbard

model with attractive interaction  $U$  and power-law hopping,

$$\mathcal{H} = - \sum_{l \neq m, \sigma}^L \left( t_{lm} \hat{c}_{l,\sigma}^\dagger \hat{c}_{m,\sigma} + \text{H.c.} \right) - \mu \sum_{l=1, \sigma}^L \left( \hat{n}_{l,\sigma} - \frac{1}{2} \right) - |U| \sum_{l=1}^L \left( \hat{n}_{l,\uparrow} - \frac{1}{2} \right) \left( \hat{n}_{l,\downarrow} - \frac{1}{2} \right), \quad (1)$$

where the fermionic annihilation operator  $\hat{c}_{l,\sigma}$  destroys an electron at site  $l$  in spin state  $\sigma (= \uparrow, \downarrow)$  and  $\hat{n}_{l,\sigma} \equiv \hat{c}_{l,\sigma}^\dagger \hat{c}_{l,\sigma}$  is the fermionic number operator. The (real) long-range hopping amplitude  $t_{lm} = t/|l-m|^\alpha$  connects sites  $l$  and  $m$ ,  $\mu$  is a uniform chemical potential corresponding to  $N$  particle per spin, and  $U$  controls the attractive interaction strength. For hopping restricted to nearest neighbors, solved exactly in [30], SC quasi-LRO dominates over the competing charge-density wave correlations, except at half-filling where both correlations are comparable. On the other hand, purely imaginary power-law hopping in a 1D *repulsive- $U$*  Hubbard model at half filling, investigated in [31] for  $\alpha = 1$  ( $d_{\text{eff}} = 2$ ), induces a Mott metal-insulator transition at a finite value of the interaction strength.

From now on we focus on the region  $|U| \gg t$  where the local attractive interaction in Eq. (1) dominates. In this regime, unpaired electrons are effectively forbidden at sufficiently low energies, and only Cooper pairs  $\hat{c}_{l,\uparrow}^\dagger \hat{c}_{l,\downarrow}^\dagger |0\rangle$  are stable configurations. We therefore project out the singly-occupied sites at order  $t/|U|$  with the unitary transformation  $\mathcal{H}_{\text{eff}} = e^{\mathcal{S}} \mathcal{H} e^{-\mathcal{S}}$ , with  $\mathcal{S} = -i (H_t^+ - H_t^-) / |U|$  and  $H_t^+ = -\sum_{l \neq m, \sigma} t_{lm} (1 - \hat{n}_{l\bar{\sigma}}) \hat{c}_{l\sigma}^\dagger \hat{c}_{m\sigma} \hat{n}_{m\bar{\sigma}}$  and  $H_t^- = -\sum_{l \neq m, \sigma} t_{lm} \hat{n}_{l,\bar{\sigma}} \hat{c}_{l\sigma}^\dagger \hat{c}_{m\sigma} (1 - \hat{n}_{m\bar{\sigma}})$ . The procedure is similar to the usual one employed to obtain the  $t$ - $J$  model [32]. Here we mention the final result,

$$\mathcal{H}_{\text{eff}} = \frac{|U|L}{4} - \mu \sum_l (\hat{n}_l - 1) + \frac{4t^2}{|U|} \sum_{l \neq m} \left[ \frac{(\hat{n}_l - 1) \hat{n}_m - \hat{\Delta}_l^\dagger \hat{\Delta}_m}{|l-m|^{2\alpha}} + \text{H.c.} \right], \quad (2)$$

which effectively is a (long-range) variant of the well-known short-range Bose-Hubbard model with hard-core bosons [3]. Here  $\hat{n}_l \equiv \hat{n}_{l,\uparrow} + \hat{n}_{l,\downarrow}$  is the total number operator at site  $l$  and  $\hat{\Delta}_l^\dagger \equiv \hat{c}_{l,\uparrow}^\dagger \hat{c}_{l,\downarrow}^\dagger$  is the creation operator for a Cooper pair at site  $l$ . The term in square brackets in the above equation arises from second-order virtual processes in the hopping  $t_{lm}$  and contains the basic ingredients leading to stabilization of the SC ground state driven by power-law hopping. Note that the coupling  $\hat{\Delta}_l^\dagger \hat{\Delta}_m$  minimizes the energy of the system by delocalizing the Cooper pairs (thus favoring a more robust SC). By contrast the density-density interaction  $(\hat{n}_l - 1) \hat{n}_m$  is strongly frustrated by power-law hopping. Therefore

the competing charge density wave phase cannot be stabilized. The crucial sign difference between these two contributions is related to the assumption of purely real hoppings  $t_{lm}$ .

We now introduce the framework of the Abelian bosonization [3]. As a first step, we take the limit of vanishing lattice parameter  $a \rightarrow 0$  in Eq. (2) and define the density  $\hat{n}_l/a \rightarrow \rho(x)$  and pair-creation  $\hat{\Delta}_l^\dagger/a \rightarrow \Delta^\dagger(x)$  operators in the continuum. We next introduce the representation [3]

$$\rho(x) = \left[ \rho_0 - \frac{\nabla \phi(x)}{\pi} \right] \sum_p e^{2ip(\pi \rho_0 x - \phi(x))} \quad (3)$$

$$\Delta(x) = \rho_0 e^{-i\theta(x)} \sum_p e^{2ip(\pi \rho_0 x - \phi(x))}, \quad (4)$$

where  $\theta(x)$  and  $\phi(x)$  are bosonic fields slowly varying on the scale of  $a$ . They satisfy the canonical commutation relations  $[\nabla \phi(x), \theta(y)] = i\pi \delta(x-y)$ . The field  $\theta(x)$  is physically related to the phase of the SC order parameter in the original system via  $\langle \Delta(x) \rangle = \langle \hat{c}_{x,\uparrow}^\dagger \hat{c}_{x,\downarrow}^\dagger \rangle \propto \langle e^{-i\theta(x)} \rangle$ , while the field  $\phi(x)$  is related to slow Cooper pair density fluctuations  $\delta \rho(x) \simeq -\nabla \phi(x) / \pi$ .

The bosonic representation (3) and (4) allows to express the low-energy part of Hamiltonian (2) as

$$\mathcal{H}_{\text{eff}} = \int dx \left[ \tilde{\mu} \frac{\nabla \phi(x)}{\pi} + \frac{uK}{2\pi} (\nabla \theta(x))^2 + \frac{u}{2\pi K} (\nabla \phi(x))^2 \right] - g \frac{\pi u \rho_0^2}{4Ka^{1-2\alpha}} \int_{|x-x'| > a} dx dx' \frac{\cos[\theta(x) - \theta(x')]}{|x-x'|^{2\alpha}}. \quad (5)$$

The first line of this equation is the Luttinger liquid model, where  $K$  is the dimensionless Luttinger parameter controlling the asymptotic decay of the correlation function  $\langle e^{i\theta(x)} e^{-i\theta(x')} \rangle \sim |x-x'|^{-1/K}$ , and  $u$  is the velocity of the 1D acoustic plasmons [3]. Physically, the product  $uK$  corresponds to the superfluid stiffness of the 1D SC and  $K/u$  is the compressibility. The dimensionless coefficient  $g$  is a non-universal quantity measuring the strength of the power-law hopping term. The numerical values of  $K$ ,  $u$  and  $g$  cannot be obtained from the bosonization procedure. Therefore we expect qualitative rather than quantitative predictions. Finally, we note that in Eq. (5) we have neglected higher harmonics  $\sim e^{2ip[\phi(x) - \phi(x')]}$  arising from the non-local density-density interaction in Eq. (2), since the field  $\phi(x)$  becomes strongly fluctuating due to frustration. Its overall effect can be accounted by a renormalization of  $K$ .

To make further progress, in what follows we employ the framework of the self-consistent harmonic approximation (SCHA) [33]. This method consists in introducing a Gaussian ansatz  $S_0 = \frac{1}{2\beta L} \sum_{\mathbf{q}} g_0^{-1}(\mathbf{q}) \theta_{\mathbf{q}}^* \theta_{\mathbf{q}}$  for the Euclidean action of the system where  $\mathbf{q} = (k, -\omega_m)$  and  $\omega_m = 2\pi T m$  is the bosonic Matsubara frequencies at

temperature  $T$  [34]. The functions  $g_0^{-1}(\mathbf{q})$  are unknown variational parameters which must be chosen to minimize the variational free energy  $F_{\text{var}} = F_0 + T \langle S - S_0 \rangle_0$ , with  $F_0$  the free energy associated to  $S_0$ , and  $S$  the action corresponding to Eq. (5). The notation  $\langle \dots \rangle_0$  stands for the average with respect to the trial action  $S_0$ . Minimizing  $F_{\text{var}}$  with respect to  $g_0(\mathbf{q})$  yields the self-consistent equation [3, 9, 11]

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u} \omega_m^2 + \frac{uK}{\pi} k^2 + \frac{2\pi u \rho_0^2}{K a^{2-2\alpha}} \int_a^L dr \frac{1 - \cos kr}{r^{2\alpha}} \times \exp \left[ -\frac{1}{\beta L} \sum_{\mathbf{q}'} (1 - \cos k'r) g_0(\mathbf{q}') \right] \quad (6)$$

where  $r = x - x'$ . In the regime  $1/2 < \alpha < 3/2$ ,  $L \rightarrow \infty$ ,  $T \rightarrow 0$ , an approximate solution of this self-consistent equation, asymptotically correct in the limit  $k \rightarrow 0$ , is given by the expression

$$g_0^{-1}(\mathbf{q}) = \frac{K}{\pi u} \omega_m^2 + \frac{uK}{\pi} k^2 + \eta |k|^{2\alpha-1}. \quad (7)$$

The last term  $\sim \eta |k|^{2\alpha-1}$  encodes the effect of power-law hopping in the system and is crucial in the rest of the analysis. Replacing (7) into (6) yields the equation for  $\eta$  [9, 11, 35]

$$\tilde{\eta} = -\frac{2\pi^2 (\rho_0 a)^2}{K^2} \Gamma(1-2\alpha) \sin(\pi\alpha) e^{-G(0)}, \quad (8)$$

where  $G(\tilde{r}) \equiv \frac{1}{2K} \int_0^\infty d\tilde{k} e^{-\tilde{k}} \cos(\tilde{k}\tilde{r}) / \sqrt{\tilde{k}^2 + \tilde{\eta} \tilde{k}^{2\alpha-1}}$ ,  $\tilde{k} \equiv ka$ ,  $\tilde{r} = r/a$ ,  $\tilde{\eta} \equiv \pi \eta a^{3-2\alpha} / uK$  and  $\Gamma(z)$  is the Euler Gamma function [36]. Note that for  $k \rightarrow 0$ , the term  $\sim \eta |k|^{2\alpha-1}$  dominates over the term  $k^2$  in Eq. (7) for  $1/2 < \alpha < 3/2$ . This is the key ingredient for the restoration of phase coherence in this region. For  $\alpha < 1/2$ , the integral in Eq. (6) becomes divergent due to the slowly decaying hopping, and consequently expression (7) is no longer a valid solution. In this regime, fluctuations around the ordered groundstate with  $k = 0$  are fully suppressed and mean-field theory becomes exact. For  $\alpha > 3/2$  the term  $k^2$  dominates for  $k \rightarrow 0$ , and the model can be effectively mapped onto a 1D SC with renormalized short-range couplings [7, 30]. Therefore,  $\alpha_c = 3/2$  is the critical value that separates the regimes of quasi-LRO from robust superconducting LRO. In order to further characterize the role of power-law hopping in 1D SC, we now compute the average of the order parameter  $\langle e^{i\theta(x)} \rangle_0 = e^{-\langle \theta^2(0) \rangle_0 / 2} = \exp[-G(0)/2]$ . In stark contrast with the short range case [3, 30] the average SC order parameter is finite for any  $1/2 < \alpha < 3/2$ . Moreover, the equal-time pair correlation function becomes  $C(r) = \langle e^{i\theta(r)} e^{d_{\text{eff}} - i\theta(0)} \rangle_0 = \exp[G(r) - G(0)]$ , which in the limit  $r \rightarrow \infty$  tends to a constant  $C(r) \approx e^{-G(0)} [1 + A/r^{3/2-\alpha} + \mathcal{O}(1/r^{3-2\alpha})]$ , with  $A > 0$ . Direct comparison of  $\langle \theta^2(0) \rangle_0$  in our case and in a short-ranged  $d_{\text{eff}}$ -dimensional results in an explicit expression

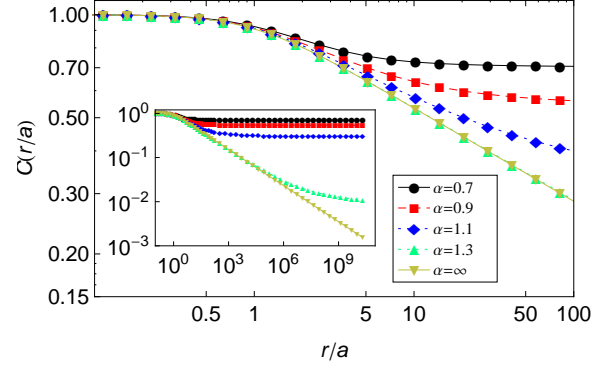


FIG. 1: (Color online) Analytic results for  $C(r/a)$  as obtained with the SCHA for different values of  $\alpha$ . In order to compare with DMRG results,  $K = 1.8$ ,  $\rho_0 a = 2N/L = 0.1803$ . For  $\alpha = 0.7$  (continuous black line), the onset of SC LRO is evident in the emergence of a plateau from  $r/a \approx 10$  ( $\xi = 1.5$ ), while for  $\alpha = 1.3$  (inset) the characteristic length of the plateau is  $\xi = 1.6 \times 10^6$ . Instead, quasi-LRO is observed for  $r < \xi$ , with effective Luttinger parameter  $K_{\text{eff}} \approx 3.7$ .

of  $d_{\text{eff}}$  as a function of  $\alpha$ ,  $d_{\text{eff}} = 2/(2\alpha - 1)$ . Therefore, for  $d_{\text{eff}} > 1$ , corresponding to  $1/2 < \alpha < 3/2$ , LRO and phase coherence are restored (see Fig. 1). This is the main result of this Letter. Finally we estimate the minimum length scale  $\xi$  at zero temperature necessary to observe LRO by equating the contributions  $|k|^{2\alpha-1}$  and  $k^2$  in Eq. (7), [37–39]

$$\xi \approx \left( \frac{uK}{\pi\eta} \right)^{\frac{1}{3-2\alpha}}, \quad (9)$$

where it is assumed that  $\{L, r\} \gg \xi$  [cf. Fig.1].

*Numerical results.*— We now test the results from the bosonization approach above by DMRG [37–39]. Power-law hopping is a challenge for many-body numerical simulations as finite size effects become much more important. In the context of DMRG also the number of basis states that must be kept increases dramatically with respect to short-range models. Moreover as the critical value  $\alpha_c = 3/2$  is approached, the crossover length scale  $\xi$  diverges [cf. Eq.(9)], and DMRG is unable to reach the LRO region. With these limitations in mind, we compute the spatial average of the pair correlation function

$$C(r) \equiv \frac{1}{L - 2l_0 - r} \sum_{l=l_0+1}^{L-l_0-r} \langle \hat{\Delta}_{l+r} \hat{\Delta}_l^\dagger \rangle, \quad (10)$$

using the DMRG, where  $\langle \dots \rangle$  stands for the average in the ground state of the microscopic model Eq. (1), and  $l_0$  is the number of sites at the end of the chain which are eliminated in order to minimize finite-size effects. In the limit  $|U|/t \gg 1$ , where amplitude fluctuations of the SC order parameter are negligible, the correlation  $C(r)$  [normalized by  $C(0)$ ] should compare to the analytical results of Fig. 1. In Fig. 2 we plot  $C(r)$  as a function

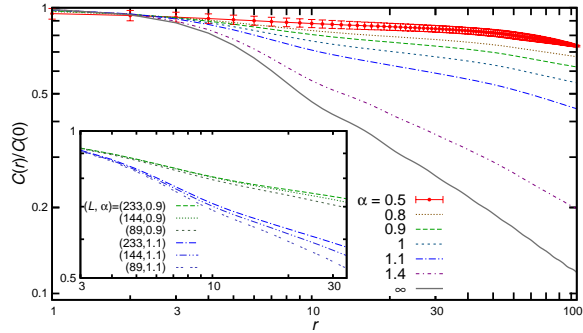


FIG. 2: (Color online)  $C(r)$  Eq. (10), computed by DMRG, for  $L = 233, N = 21, U/t = -20, l_0 = L/4$  and different  $\alpha$ 's. In agreement with the bosonization results, LRO is clearly observed up to  $\alpha \sim 1$ . For larger  $\alpha$ , the crossover length [cf. Eq. 9] to observe LRO is larger than the maximum size accessible by DMRG techniques. The standard deviation (obtained by taking the spatial average) is shown only for  $\alpha = 0.5$ . A finite size scaling analysis with almost constant  $N/L$  of  $C(r)$  (inset) in the region  $\alpha \sim 1$  indicates that the decay with the distance becomes slower as we increase the system size. It is therefore plausible that  $C(r)$  will eventually flatten out as the  $L \rightarrow \infty$  is approached.

of  $r$  for different values of  $\alpha$  and  $|U|/t = 20$ . The numerical results are consistent with the previous analytical calculation. LRO seems to occur for  $\alpha \lesssim 1.1$  though as  $\alpha$  increases a very slow downward trend is observed. In order to further clarify whether the very slow decay observed for  $\alpha \sim 1$  disappears in the thermodynamic limit we carry out a finite size scaling analysis. The results, depicted in Fig. 2 (inset), show that the decay becomes slower as the system size increases. This is a strong indication that true LRO also occurs for  $1 < \alpha < 3/2$ . Qualitatively similar results are obtained for other filling factors and interaction strength provided that  $|U|/t \gg 1$ , in order to avoid that  $\xi > L$ . For  $\alpha > 1.1$ , also in agreement with the analytical calculation, only quasi-LRO is observed for the sizes accessible by DMRG techniques. Much larger sizes  $L$  are needed to observe LRO. In conclusion, DMRG results indicate that for  $|U|/t \gg 1$  and  $\alpha \leq 1.1$  LRO is restored by power-law hopping.

*Implications for transport.*— While a detailed study of the transport properties is beyond the scope of the present Letter, it is clear that the last term in Eq. (5) will have strong effects on the QPS/anti-QPS interaction, which determines the resistivity of the 1D superfluid [7, 40]. In our case, we expect the attractive interaction between a QPS located at  $\mathbf{r}_i = (x_i, u\tau_i)$ , and an anti-QPS located at  $\mathbf{r}_j = (x_j, u\tau_j)$  to grow faster than  $\ln(|\mathbf{r}_i - \mathbf{r}_j|/a)$ , the typical interaction in a short-ranged 1D SC [7, 40]. Therefore, in our case the QPS/anti-QPS pair should be tightly bound and consequently we expect the resistivity of the system to approach to zero much faster than the short-ranged case as  $T \rightarrow 0$  and  $L \rightarrow \infty$ .

A more detailed study of the transport properties of the wire is currently in progress [41].

*Application to quantum spin chains.*— These findings not only provide novel insights into the nature of the superconducting state in low spatial dimensions, but are also of interest to quantum magnetism. Using the Anderson pseudo-spin representation [42]  $\hat{n}_l \rightarrow \hat{S}_l^z + 1/2$ ,  $\hat{\Delta}_l^\dagger \rightarrow \hat{S}_l^+$ , Eq. (2) can be mapped onto a spin-1/2  $XXZ$  chain with an effective Zeeman field along the  $z$ -axis, and long-range antiferromagnetic (ferromagnetic) couplings along the  $z$ -axis ( $xy$ -plane). In this form, we can see immediately that the long-range nature of the exchange couplings will induce frustration along the  $z$ -axis, but will favor ferromagnetic LRO in the  $xy$ -plane. As mentioned above, it recently became experimentally feasible to engineer a broad range  $0 \leq \alpha \leq 3$  of power-law decaying interactions in cold trapped-ion systems that mimic quantum spin chains [28, 29]. Our results are therefore relevant in the description of the rich phase diagram of these systems.

In conclusion, we have investigated the 1D attractive- $U$  Hubbard model with an algebraic hopping by means of Abelian bosonization and DMRG techniques. Results from both approaches are consistent: at  $T = 0$ , true LRO is recovered for  $\alpha < 3/2$ , corresponding to an effective dimensionality  $d_{\text{eff}} > 1$ . However, for  $\alpha > 1.1$  LRO cannot be observed numerically because it requires system sizes far larger than those accessible by DMRG. The robustness of superconductivity in  $d_{\text{eff}} > 1$  paves also the way to boost superconductivity by shell effects [22] and other coherence effects important in low-dimensional and nanoscale SCs. Our results are of interest in other problems beyond superconductivity, especially in 1D quantum magnetism, where the resulting phase diagram can be investigated experimentally in trapped-ion systems.

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